and nonrepeatability of QCM positioning when removed from and reassembled into the apparatus. Apparatus symmetry was verified during checkout to better than ±0.5% using identical 10-MHz QCMs at 85 K, and water as a molecular source. The QCM dimensions are known to better than ±0.13 mm, and so the locations of their crystals relative to their support struts were known to this accuracy. Because of apparatus complexity, the radial distance from the cell orifice to the QCM crystals was difficult to measure to better than ±0.5 mm, which, for a 15-cm nominal radial distance, corresponds to a view factor error of about ±0.7%. The resulting error in calculated relative view factor should be smaller than this, since the radial distance error would affect the view factor for both QCMs. Angular position errors of the QCMs relative to the apparatus centerline are included in the ±0.5% symmetry error. The total error in the calculated relative view factor of 1.053 should be on the order of about ±1.0%. Since the accuracy of the comparison is directly proportional to the accuracy of the relative view factor, it is concluded that, for both 15-A and 15-B, the measured values of S_{15}/S_{10} agree with the theoretical ratio within this range of experimental uncertainty. This compares to the approximately ±2% agreement between calibrated and theoretical mass sensitivity reported in the literature.[6]

The difference between S_{15}/S_{10} for tests 15-A and 15-B is almost certainly because of a difference between actual and calculated view factors from differences in the actual positions of the QCMs after assembly into the apparatus. The two 15-MHz QCMs are geometrically identical, but variations in position relative to the support struts could have occurred between successive assemblies because of excessive bolt hole clearances at the attachment interfaces. A 0.7% difference in view factor would correspond to a quite-probable repositioning variation of 0.5 mm.

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Analytical Expression for a Concentric-Cylinder Radiation View Factor

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Nomenclature

A_i = area of cylinder i
L = distance between the bases of (finite length) rings
r_i = radius of cylinder i
S = length of the ray between a point on one ring and a point on the other ring
z_i = axial coordinate in ring i
z = distance between (differential) rings
α = $(z^2/2r_1r_2) + \frac{1}{2}(\phi_1 + \phi_2)$
β_i = angle between surface-normal of ring i and the ray S that connects the rings
δ_i = length of ring i
θ = azimuthal angle, on the plane of the shell ring
ϕ_1 = r_1/r_2
ϕ_2 = $1/\phi_1$
φ = azimuthal angle, on the plane of the tube ring

Subscripts
1 = tube, i.e., the inner cylinder
2 = shell, i.e., the outer cylinder

Introduction

RADIATION is a major mode of heat transfer in high-temperature processes; the radiation view factor (also known as configuration factor, shape factor, angle factor, etc.) is a key element of the underlying heat balances. Software packages exist (e.g., FACET, VIEW) to find radiation view factors numerically for any complicated configuration, taking obstructions into account.[1] Nevertheless, simple and accurate analytical expressions for the view factor are always of interest. The shell and tube geometry is common to rockets, reactors, heat exchangers, and tubular furnaces. Hence, it can be expected that view factors for this geometry can be found in the literature. Indeed, a literature search revealed several sources.[2–7]

The comprehensive report by Leuenberger and Person[2] includes an expression for the differential view factor between a ring on the tube exterior and the finite annular space on the end plug between the tube and the shell, but none for the differential view factor between a differential ring on the tube exterior and a differential ring on the shell interior. Finding or deriving the latter became the objective of this work.

The expressions derived by Rea[3] or Shukla and Ghosh[4] are for finite view factors between two coaxial cylinders of different lengths that are displaced from one another; the expressions compiled/derived by Brockmann[5] are also for finite view factors between two concentric cylinders of equal lengths. These view factors, which apply to finite cylinders that are end-capped, not to differential ring elements on two concentric

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cylinders, are certainly related to the differential ring-to-ring view factor, but probably not in a way that easily leads to a closed-form expression.

The other two sources[6,7] seem to provide the view factor sought here. The following definition brings out the difference between the two approaches.

Definition

The view factor between a finite length ring on the tube and a finite length ring on the shell is defined as

$$F_{1\to 2} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi S^2}\, dA_2\, dA_1 \qquad (1)$$

or

$$F_{1\to 2} = \frac{1}{2\pi r_1 \delta_1}\int_0^{\delta_1}\int_0^{2\pi}\int_0^{\delta_2} \times \left(\frac{2}{\pi}\int_{\theta_{\min}}^{\theta_{\max}} \frac{\cos\beta_1\cos\beta_2}{S^2}\, r_2\, d\theta\, dz_2\right) r_1\, d\varphi\, dz_1 \qquad (2)$$

The first (i.e., innermost) integration is around θ; the coefficient 2 represents the symmetry about the azimuthal angle.

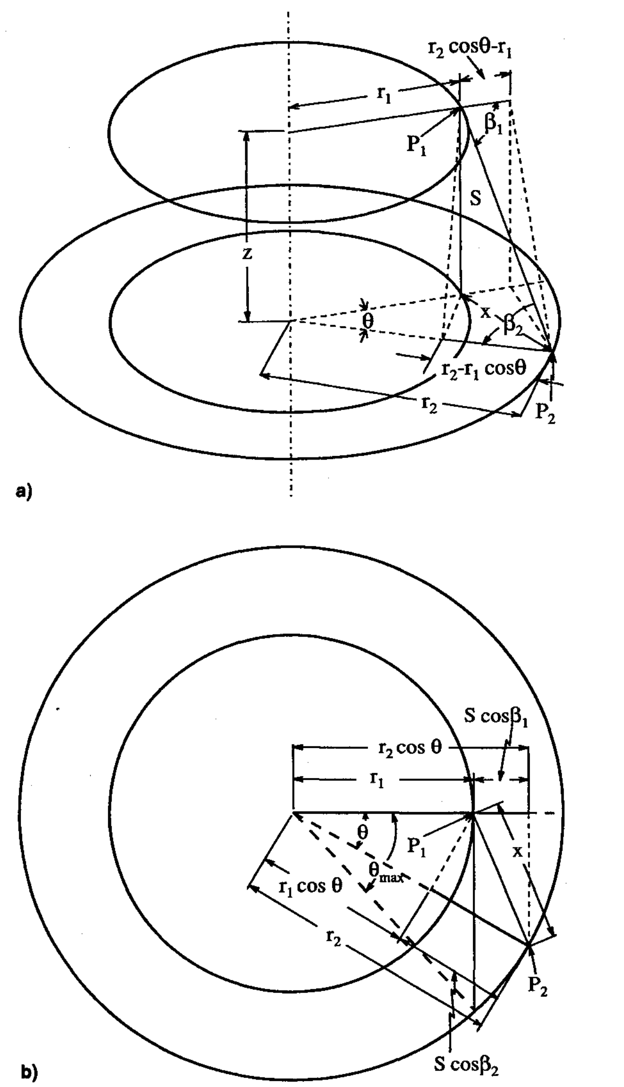

Fig. 1 Differential rings on concentric cylinders: a) isometric and b) plan views.

In physical terms (see Fig. 1), the limits $\theta_{\min}$ and $\theta_{\max}$ define how much of the shell ring can be seen from the radiation source-point on the tube ring.

The second integration, about dz_2, is over the length δ_2 of the shell ring. The quantity inside the parentheses is defined as $dF_{d1\to d2}$, the differential view factor between the tube ring and the shell ring.

The third integration, about $d\varphi$, corresponds to moving the source-point around the tube ring's periphery. When there are no obstructions between tube and shell, all source-points on the tube see the shell ring to the same extent. Accordingly, the third integration simply leads to a multiplication by 2π.

The fourth integration, about dz_1, is over the length δ_1 of the tube ring.

The order of integration should not affect the result, but is a matter of physical or mathematical convenience. In fact, the two sources cited previously differ in the order of integration.

By the Reid and Tennant[6] approach, one would first carry out the integrations (analytically) over dz_1, dz_2, and $d\varphi$. The resulting expression can be considered as 2π times the view factor between a strip on the tube ring and a strip on the shell ring. While this expression would be useful when the temperature varies around the periphery of either ring, it is not the differential view factor between the two rings. Reid and Tennant carried out the remaining integration (numerically) with respect to the azimuthal angle $d\theta$, obtained a view factor for finite length rings, and plotted the factor functions for values of r_2/r_1 = 1.5, 3, and 5. As discussed later, these plots are useful in verifying the present derivation.

On the other hand, Modest[7] integrated first (analytically) azimuthally; i.e., over $d\theta$. The resulting expression from that work is more general than is needed here because it is for any two arbitrarily shaped axisymmetric bodies, e.g., the plasma chamber of the Next European Torus (NET) fusion reactor.

One could use Modest's final equation and substitute in it the specifics of the current geometry, or one could (re)derive the view factor expression for the current geometry using Modest's approach. We chose the latter option. The derivation is summarized next. (Detailed derivation is available upon request.)

Derivation

As noted previously, the differential view factor for radiation from a differential ring element on the tube (subscript 1) to a differential ring on the shell (subscript 2) is defined as

$$dF_{d1\to d2} = \frac{2}{\pi}\int_{\theta_{\min}}^{\theta_{\max}} \frac{\cos\beta_1\cos\beta_2}{S^2}\, r_2\, d\theta\, dz_2 \qquad (3)$$

Using trigonometric identities (see Fig. 1), the expression can be converted to

$$dF_{d1\to d2} = \frac{-dz_2}{2\pi r_1}\int_0^{\cos^{-1}\phi_1} \frac{(\phi_1 - \cos\theta)(\phi_2 - \cos\theta)}{(\alpha - \cos\theta)^2}\, d\theta \qquad (4)$$

The integration with respect to $d\theta$ can be done analytically (making use of Ref. 8) to get

$$dF_{d1\to d2} = \frac{dz_2}{2\pi r_1}\left[2\,\frac{(\alpha^3 + \phi_1 + \phi_2 - 3\alpha)}{(\alpha^2 - 1)^{3/2}} \times \tan^{-1}\sqrt{\frac{(\alpha+1)(1-\phi_1)}{(\alpha-1)(1+\phi_1)}} - \frac{(\alpha - \phi_2)\sqrt{(1-\phi_1^2)}}{(\alpha^2 - 1)} - \cos^{-1}\phi_1\right] \qquad (5)$$

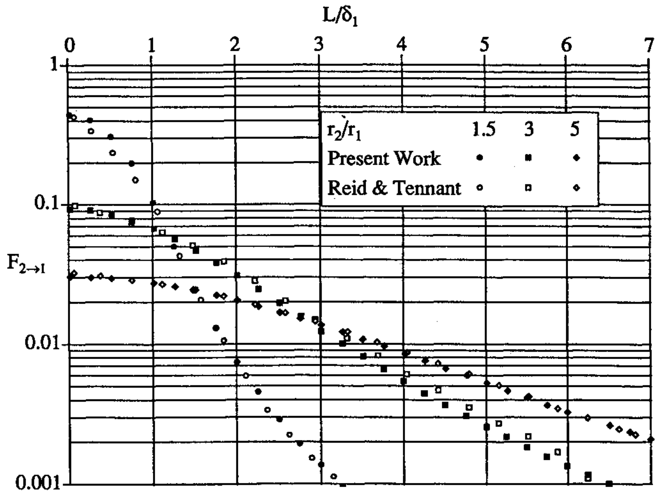

Fig. 2 Finite area shell ring to tube ring view factors for $\delta_1/r_1 = 1$.

Verification

The view factor expression was checked for accuracy in two ways:

1) After some algebra, it was verified that the integrand equals the one used by Reid and Tennant.[6]

2) Next, the differential ring-to-ring view factor ($dF_{d1\to d2}$) was numerically integrated to obtain values for the finite view factor ($F_{2\to 1}$) between finite length rings:

$$F_{2\to 1} = \frac{r_1}{r_2\delta_2}\int_0^{\delta_1}\int_0^{\delta_2}(dF_{d1\to d2})\,dz_1 \tag{6}$$

Because Reid and Tennant's results are plotted rather than tabulated, a scanner was used to read off values that could be (re)plotted against the current results (Fig. 2). The agreement is excellent.

It can be concluded that the derived expression for the annular ring-to-ring differential view factor is correct. We believe that this work fills a need since, surprisingly, a simple closed-form expression for this important view factor is not included in standard compilations (e.g., Refs. 9 and 10).

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Novel Stokesmeter

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Introduction

THE identification of radiation in terms of its polarization characteristics is becoming important in various areas of science and engineering.[1-4] In particular, radiation transport involving reflection, transmission, absorption, and scattering may be very dependent on the polarization traits of that radiation. The general description of the polarization characteristics is based on the polarization ellipse. While conceptually convenient, in that only one equation is used to describe the state of the polarized radiation, this concept is not 100% sufficient. The inadequacy is because of the fact that the ellipse can be used to describe only completely polarized radiation. As an alternative, the radiation field may be described by average characteristics that can be measured.

A convenient procedure for describing polarized radiation is the Stokes parameters that may be used for completely, partially, and unpolarized radiation. The main purpose of this note is to present a description of an inexpensive device that may be used to measure these Stokes polarization parameters. In addition, a novel degenerate form of the Stokes parameters is introduced.

Stokes Vectors

As a review of the Stokes polarization parameters, recall that these quantities are set up in a vector form.[5-10] That is

$$\bar{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{bmatrix} E_{ox}^2 + E_{oy}^2 \\ E_{ox}^2 - E_{oy}^2 \\ 2E_{ox}E_{oy}\cos(\delta) \\ 2E_{ox}E_{oy}\sin(\delta) \end{bmatrix} \tag{1a}$$

$$= S_0 \begin{bmatrix} 1 \\ \cos(2\alpha) \\ \sin(2\alpha)\cos(\delta) \\ \sin(2\alpha)\sin(\delta) \end{bmatrix} = S_0 \begin{bmatrix} 1 \\ \cos(2\chi)\cos(2\psi) \\ \cos(2\chi)\sin(2\psi) \\ \sin(2\chi) \end{bmatrix} \tag{1b}$$

In the first representation of Eq. (1a), S_0 is the intensity of the beam radiant energy, S_1 describes the linear horizontal or vertical polarization present, S_2 describes the presence of linear polarization at $\pi/4$ or $-\pi/4$, and S_3 describes the right or left circular polarization in the beam. Further

$$S_0^2 \geq S_1^2 + S_2^2 + S_3^2 \tag{2}$$

where the equal sign is used when the beam is completely polarized. If the beam has an unpolarized component the degree of polarization is defined as

$$p = \sqrt{S_1^2 + S_2^2 + S_3^2}/S_0^2 \tag{3}$$

In the second representation of Eq. (1a), E_{ox} and E_{oy} are the maximum amplitudes of the electric field vectors and δ, the

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